NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2410

ANALYTICAL INVESTIGATION OF FULLY DEVELOPED LAMINAR FLOW IN TUBES WITH HEAT TRANSFER WITH FLUID PROPERTIES VARIABLE ALONG THE RADIUS

By Robert G. Deissler

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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SUMMARY

Relations were analytically obtained for the prediction of radial distributions of velocity and temperature for fully developed laminar flow of gases and of liquid metals in tubes with fluid properties variable along the radius. The relations are applicable to both heating and cooling of the fluid. By use of the relations for velocity and temperature distributions, relations were obtained among Nusselt number, friction parameter, and ratio of wall to bulk temperature.

The Nusselt number and friction parameter were found to be independent of Reynolds number and Prandtl number. The effects of ratio of wall to bulk temperature on Nusselt number and friction parameter could be eliminated by evaluating the fluid properties at specified temperatures in the fluid.

INTRODUCTION

Most of the analyses of laminar flow and heat transfer in tubes, for instance the uniform-wall-temperature solution given in reference 1, have been carried out under the assumption that the fluid properties do not vary with temperature. Somewhat more work on variable fluid properties has been done for the laminar boundary layer of a flat plate. (See, for instance, references 2 and 3). In reference 4, velocity distributions for laminar flow in tubes with variable viscosity and some approximate relations for temperature distributions and heat-transfer coefficients are given.

In the analytical investigation reported herein, which was conducted in the NACA Lewis laboratory, both velocity and temperature distributions are obtained for fully developed laminar flow in tubes of gases and of liquid metals with variable fluid properties. These relations are applicable to both heating and cooling of the fluid. By use of these velocity and temperature distributions, Nusselt numbers and friction parameters are obtained. The results should be applicable to flow at points where the ratio of distance from entrance to diameter of the tube is on the order of one-twentieth of the Reynolds number or greater.

SYMBOLS

The following symbols are used in the report:

c_p	specific heat of fluid at constant pressure, (Btu)/(lb)(°F)
D	inside diameter of tube, (ft)
d	exponent, value of which depends on variation of viscosity of fluid with temperature
f	friction factor, $2\tau_0/(\rho u_b^2)$
f_x	friction factor with density evaluated at t_x
f_0	friction factor with density evaluated at wall temperature
g	acceleration due to gravity, (32.2 ft/sec^2)
h	heat-transfer coefficient, $q_0/(t_0 - t_b)$, (Btu)/(sec)(sq ft)(°F)
k	thermal conductivity of fluid, (Btu)(ft)/(sec)(sq ft)(°F)
k_b	thermal conductivity of fluid evaluated at t_b, (Btu)(ft)/(sec)(sq ft)(°F)
k_0	thermal conductivity of fluid evaluated at t_0, (Btu)(ft)/(sec)(sq ft)(°F)
Nu	Nusselt number, hD/k
Nu_b	Nusselt number with conductivity evaluated at t_b
Nu_x	Nusselt number with conductivity evaluated at t_x
Nu_0	Nusselt number with conductivity evaluated at t_0
Pr	Prandtl number, $c_p g\mu/k$

q rate of heat transfer toward tube center per unit area, (Btu)/(sec)(sq ft))

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q_0 rate of heat transfer at wall toward tube center per unit area, (Btu)/(sec)(sq ft))

q' dimensionless heat-transfer parameter, q/q_0

r radius, distance from tube center, (ft)

r_0 inside tube radius, (ft)

r' dimensionless radius parameter, r/r_0

Re Reynolds number, $\rho u_b D/\mu$

Re_x Reynolds number with density and viscosity evaluated at t_x

Re_0 Reynolds number with density and viscosity evaluated at wall

t absolute temperature, ($^{\circ}$R)

t_b bulk or average static temperature of fluid at cross section of tube, ($^{\circ}$R)

t_x temperature in fluid, $x(t_0 - t_b) + t_b$, ($^{\circ}$R)

t_0 absolute wall temperature, ($^{\circ}$R)

t' dimensionless temperature parameter, t/t_0

t_b' dimensionless bulk-temperature parameter, t_b/t_0

u velocity parallel to axis of tube, (ft/sec)

u_b bulk or average velocity at cross section of tube, (ft/sec)

u' dimensionless velocity parameter, $\mu_0 u/(\tau_0 r_0)$

u_b' dimensionless bulk velocity parameter, $\mu_0 u_b/(\tau_0 r_0)$

x number

z distance from tube entrance, (ft)

λ dimensionless parameter, $k_0 t_0/(q_0 r_0)$

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δ dimensionless friction parameter, $\tau_0 r_0/(u_b \mu)$

δ_0 dimensionless friction parameter with viscosity evaluated at t_0

δ_x dimensionless friction parameter with viscosity evaluated at t_x

μ absolute viscosity of fluid, (lb-sec/sq ft)

μ_b absolute viscosity of fluid evaluated at t_b, (lb-sec/sq ft)

μ_0 absolute viscosity of fluid at wall, (lb-sec/sq ft)

ρ mass density, (lb-sec^2/ft^4)

ρ_b bulk or average density at cross section of tube, (lb-sec^2/ft^4)

ρ_0 mass density of fluid at wall, (lb-sec^2/sq ft^4)

τ shear stress in fluid, (lb/sq ft)

τ_0 shear stress in fluid at wall, (lb/sq ft)

ANALYSIS

In the present analysis, equations for the velocity and temperature distributions in a laminar stream of gases or of liquid metals through a tube are derived for the fully developed boundary layer. From these equations, relations for the Nusselt number and friction factor are obtained.

Flow of Gases

Assumptions. - The analysis for gases applies when the following assumptions hold:

1. The velocity u at any given distance from the wall is independent of distance along the tube. This assumption holds for fully developed flow when the rate of density change in the direction along the tube is small. The effect of density change becomes less important as the velocity is increased. Laminar flow can exist at these velocities if the tube diameter is small.

2. The temperature difference $t_0 - t$ at any distance from the wall is independent of distance along the tube. This condition holds for fully developed temperature distributions with uniform longitudinal

distribution of heat transfer in the tube when the fluid properties are constant and holds for variable properties when the heat-transfer rate varies slightly along the tube.

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3. Frictional heating effects can be neglected. These effects are small when the Mach number for the flow is small compared with unity; the Mach number is generally small when laminar flow through a tube exists.

4. The viscosity, the thermal conductivity, and the density vary with temperature; the specific heat and the Prandtl number are constant. The variations with temperature of the specific heat and the Prandtl number of gases are of a lower order of magnitude than the variation of viscosity, thermal conductivity, and density.

5. Heat transfer by conduction in the gas in the direction along the tube can be neglected. This heat transfer is generally small compared with that transferred by convection along the tube except for very small velocities.

<u>Velocity distributions.</u> - For laminar flow the shear stress is proportional to the velocity gradient, or

$$\tau = -\mu \frac{du}{dr} \tag{1}$$

The relation for the variation of shear stress with radius for unaccelerated flow (assumption 1) is obtained by equating the shear forces to the pressure forces acting on a cylinder of fluid of arbitrary radius and differential length (reference 5). This relation gives

$$\tau = \tau_0 \frac{r}{r_0} \tag{2}$$

From equations (1) and (2)

$$\tau_0 \frac{r}{r_0} = -\mu \frac{du}{dr} \tag{3}$$

In order to convert this equation to dimensionless form, the following quantities are introduced:

$$u' \equiv \frac{\mu_0 u}{\tau_0 r_0} \tag{4}$$

$$r' \equiv \frac{r}{r_0} \tag{5}$$

Then

$$\frac{du'}{dr'} = \frac{\mu_0}{\tau_0}\frac{du}{dr} \tag{6}$$

Equation (3) becomes, when written in dimensionless form, 2154

$$r' = -\frac{\mu}{\mu_0}\frac{du'}{dr'} \tag{7}$$

The variation of the viscosity of a gas or liquid metal with temperature can be shown from viscosity data to be given by an equation of the form

$$\frac{\mu}{\mu_0} = \left(\frac{t}{t_0}\right)^d = t'^d \tag{8}$$

Substituting this value for μ/μ_0 in equation (7) and separating variables results in

$$du' = -\frac{r'}{t'^d}\,dr'$$

Integration of this equation between a point in the fluid and the wall results in

$$u' = \int_{r'}^{1} \frac{r'}{t'^d}\,dr' \tag{9}$$

In order to solve this equation, the relation between t' and r' must be obtained. This relation is obtained in the following section.

<u>Temperature distributions.</u> - For obtaining temperature distributions, a heat balance is first written for an annulus of fluid with inside radius r, outside radius $r + dr$, and length dz. The heat entering the outer cylinder of the annulus is $2\pi(r + dr)(q + dq)dz$ and the heat leaving the inner cylinder is $2\pi\ r\ q\ dz$. The net heat transferred into the annulus through the cylinders is, if differentials of higher order than the first are neglected, $2\pi\ dz(rdq + qdr)$ or $2\pi\left[\partial(rq)/\partial r\right]dr\ dz$. The heat transferred through the ends of the annulus by conduction is neglected (assumption 5). The heat picked up by fluid passing through the annulus is $2\pi\ r\ dr\ \rho gu\ c_p\ dt$ or $2\pi\ r\ dr\ dz\ \rho g\ u\ c_p\ \partial t/\partial z$. The heat balance can then be written as

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$$\frac{\partial(rq)}{\partial r} = r\rho g\ u\ c_p \frac{\partial t}{\partial z} \tag{10}$$

The heat balance for the whole cross section of the tube is

$$2\pi\ r_0\ q_0 = \pi\ r_0^2\ \rho_b\ g\ u_b\ c_p\ \partial t_b/\partial z$$

or

$$2q_0 = r_0\ \rho_b\ g\ u_b\ c_p\ \partial t_b/\partial z \tag{11}$$

From assumption 2,

$$\frac{\partial(t_0 - t)}{\partial z} = 0$$

or

$$\frac{\partial t_0}{\partial z} = \frac{\partial t}{\partial z} = \frac{\partial t_b}{\partial z} \tag{12}$$

Dividing equation (10) by equation (11) and using condition (12) and constant c_p (assumption 4) give

$$\frac{1}{2}\frac{\partial(r'\ q/q_0)}{\partial r'} = r'\ \frac{\rho}{\rho_b}\frac{u}{u_b} \tag{13}$$

The static pressure is uniform across the tube. From the perfect gas law,

$$\frac{\rho}{\rho_b} = \frac{t_b}{t} \tag{14}$$

Substituting equation (14) in equation (13) and integrating produce

$$\int_{r'q'}^{1} d(r'q') = 2\int_{r'}^{1} r'\ \frac{t_b'}{t'}\frac{u'}{u_b'}\, dr'$$

or

$$q' = \frac{1}{r'} - \frac{2}{r'}\int_{r'}^{1} r'\ \frac{t_b'}{t'}\frac{u'}{u_b'}\, dr' \tag{15}$$

Another expression for q' is obtained as follows: The law for heat conduction is

$$q = k \frac{dt}{dr} \tag{16}$$

Equation (16) can be written in dimensionless form as

$$q' = \lambda \frac{k}{k_0} \frac{dt'}{dr'} \tag{17}$$

where

$$\lambda \equiv \frac{k_0 t_0}{q_0 r_0} \tag{18}$$

With the assumptions of constant Prandtl number and constant specific heat (assumption 4), the law for the variation with temperature of the conductivity must be the same as the law for variation with temperature of the viscosity, or

$$\frac{k}{k_0} = \left(\frac{t}{t_0}\right)^d = t'^d \tag{19}$$

Substitution of equation (19) into equation (17) gives

$$q' = \lambda t'^d \frac{dt'}{dr'} \tag{20}$$

Equating equation (15) to equation (20) gives

$$\frac{1}{r'} - \frac{2}{r'} \int_{r'}^{1} r' \frac{t_b'}{t'} \frac{u'}{u_b'} dr' = \lambda t'^d \frac{dt'}{dr'}$$

This equation can be integrated to give

$$1 - t' = \frac{1}{\lambda} \int_{r'}^{1} \frac{1}{t'^d} \left(\frac{1}{r'} - \frac{2}{r'} \frac{t_b'}{u_b'} \int_{r'}^{1} \frac{r'u'}{t'} dr'\right) dr' \tag{21}$$

In order to solve equation (21), t_b', u_b', and u' must be obtained in terms of t' and r'.

$$t_b = \frac{\int_0^{r_0} t\rho u \ rdr}{\int_0^{r_0} \rho u \ rdr} \tag{22}$$

or

$$t_b' = \frac{\int_0^1 \frac{t}{t_0} \frac{\rho}{\rho_0} u'r'dr'}{\int_0^1 \frac{\rho}{\rho_0} u'r'dr'} = \frac{\int_0^1 u'r'dr'}{\int_0^1 \frac{u'r'}{t'} dr'} \tag{23}$$

Similarly,

$$u_b = \frac{\int_0^{r_0} ur \ dr}{\int_0^{r_0} r \ dr} = \frac{2}{r_0^2} \int_0^{r_0} ur \ dr \tag{24}$$

or

$$u_b' = 2\int_0^1 u'r' \ dr' \tag{25}$$

From equations (23) and (25)

$$\frac{t_b'}{u_b'} = \frac{1}{2} \frac{1}{\int_0^1 \frac{u'r'}{t'} dr'} \tag{26}$$

When equation (26) and the value for u' from equation (9) are substituted in equation (21), there results

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$$t' = 1 - \frac{1}{\lambda}\int_{r'}^{1} \frac{1}{t'^{d}} \left(\frac{1}{r'} - \frac{1}{r'} \frac{\displaystyle\int_{r'}^{1} \frac{\displaystyle\int_{r'}^{1} \frac{r'}{t'^{d}}\,dr'}{t'}\, r'dr'}{\displaystyle\int_{0}^{1} \frac{\displaystyle\int_{r'}^{1} \frac{r'}{t'^{d}}\,dr'}{t'}\, r'dr'} \right) dr' \qquad (27)$$

Equation (27) gives the relation between t' and r' for various values of λ for laminar flow of gases. The equation can be solved by the method of iteration; that is, an assumed relation between t' and r' is substituted into the right-hand side of the equation and new values are calculated by solving the equation. The new values are then substituted into the equation and the process is repeated. After the relation between t' and r' for each value of λ has been obtained from equation (27), the relation between u' and r' can be obtained from equation (9).

Flow of Liquid Metals

Assumptions. - Assumptions 1, 2, 3, and 5, which were used in the analysis of laminar flow of gases, are also used in the analysis of laminar flow of liquid metals in tubes. Assumption 4 is altered, however, inasmuch as the variations with temperature of the density and the thermal conductivity for most liquid metals are small compared with the variations of viscosity. In the present analysis, it is assumed that only the viscosity varies with temperature.

Velocity distributions. - In the derivation of equation (9) for velocity distributions in gas flow, variable density and thermal conductivity did not enter the analysis. Equation (9) therefore applies also to liquid metals.

Temperature distributions. - The analysis of temperature distributions for liquid metals is the same as that for gases except that ρ/ρ_b in equation (13) and k/k_0 in equation (17) are both set equal to 1. The equation for temperature distributions is, with these alterations,

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$$t' = 1 - \frac{1}{\lambda}\int_{r'}^{1}\left[\frac{1}{r'} - \frac{1}{r'}\,\frac{\displaystyle\int_{r'}^{1}\left(\int_{r'}^{1}\frac{r'}{t'^{d}}\,dr'\right)r'dr'}{\displaystyle\int_{0}^{1}\left(\int_{r'}^{1}\frac{r'}{t'^{d}}\,dr'\right)r'dr'}\right]dr' \tag{28}$$

This equation can be solved in the same way as equation (27). The dimensionless bulk temperature t_b' for liquid metals is calculated from

$$t_b' = \frac{\displaystyle\int_0^1 t'u'r'dr'}{\displaystyle\int_0^1 u'r'dr'} \tag{29}$$

rather than from equation (23), which was used for gases.

Nusselt Numbers and Friction Parameters

Nusselt numbers. - The Nusselt number with the conductivity evaluated at the wall temperature is defined as

$$Nu_0 \equiv \frac{hD}{k_0} \tag{30}$$

where

$$h \equiv \frac{q_0}{t_0 - t_b} \tag{31}$$

From equations (30) and (31) and the definition of λ in equation (18),

$$Nu_0 = \frac{2}{\lambda(1 - t_b')} \tag{32}$$

where t_b' is obtained from equations (23), (9), and (27) for gases or from equations (28), (9), and (29) for liquid metals.

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Similarly the Nusselt number with the conductivity evaluated at the bulk temperature is

$$Nu_b = \frac{2}{\lambda(1 - t_b')\ t_b'^{d}} \tag{33}$$

An arbitrary temperature in the fluid t_x is defined by

$$t_x \equiv t_b + x(t_0 - t_b) \tag{34}$$

or

$$\frac{t_x}{t_0} = t_b' + x(1 - t_b') \tag{35}$$

where x is some number. The Nusselt number with the conductivity evaluated at t_x is

$$Nu_x = \frac{2}{\lambda(1 - t_b')(t_x/t_0)^d} \tag{36}$$

Friction parameters. - The friction parameter with the viscosity evaluated at the wall temperature is defined as

$$\delta_0 \equiv \frac{\tau_0 r_0}{u_b \mu_0} \tag{37}$$

or

$$\delta_0 = \frac{1}{u_b'} \tag{38}$$

where u_b' is calculated from equations (25), (9), and (27) or (28). The friction parameter with the viscosity evaluated at any temperature t_x is, then

$$\delta_x = \frac{1}{u_b'(t_x/t_0)^d} \tag{39}$$

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The conventional friction factor with density evaluated at the wall f_0 can be shown, from the definitions of f_0, δ_0, and Re_0, to be given by

$$f_0 = \frac{4\delta_0}{Re_0} \tag{40}$$

or

$$f_x = \frac{4\delta_x}{Re_x} \tag{41}$$

Conventional Velocity- and Temperature-Distribution Parameters

The conventional velocity-distribution parameter u/u_b can be written as

$$\frac{u}{u_b} = \frac{u'}{u_b'} \tag{42}$$

where u' is calculated from equation (9) and u_b' is calculated from equations (25), (9), and (27) for gases, or equations (25), (9), and (28) for liquid metals.

Similarly the temperature-distribution parameter $(t_0-t)/(t_0-t_b)$ can be written as

$$\frac{t_0-t}{t_0-t_b} = \frac{1-t'}{1-t_b'} \tag{43}$$

where t' is calculated from equation (27) for gases or equation (28) for liquid metals, and t_b' is calculated from equation (23) or (29).

Special Case of Uniform Fluid Properties

Equations (9) and (27) or (28) can be integrated for the special case where the fluid properties are constant by setting the value of t' on the right-hand sides of these equations equal to 1. The results of these integrations are

$$u' = \frac{1}{2}(1 - r'^2) \tag{44}$$

and

$$t' = 1 - \frac{1}{\lambda}\left(\frac{3}{4} - r'^2 + \frac{r'^4}{4}\right) \tag{45}$$

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Also, u_b' is found by integration of equation (25) to be

$$u_b' = \frac{1}{4} \tag{46}$$

and, from equation (29),

$$t_b' = 1 - \frac{11}{24}\frac{1}{\lambda} \tag{47}$$

From equations (32) and (47), the Nusselt number is

$$Nu = \frac{48}{11} \tag{48}$$

From equations (38) and (46)

$$\delta = 4 \tag{49}$$

and, from equations (40) and (49), the friction factor is

$$f = \frac{16}{Re} \tag{50}$$

RESULTS AND DISCUSSION

Laminar Flow of Gases

Velocity and temperature distributions. - Velocity and temperature distributions for fully developed laminar flow of gases in tubes with variable fluid properties are shown in figures 1 and 2, respectively. The profiles were calculated from equations (42) and (43), where the various quantities in the equations were calculated as indicated after the equations. The exponent d was found from viscosity data to have an average value of 0.68 for air and most common gases for temperatures between 0° and 2000° F.

It is seen from figure 1 that heat addition to the gas sharpened the peak of the velocity profile in the central portion of the tube, whereas heat extraction causes a flattening of the profile at the center of the tube. These results agree qualitatively with the sketch given in reference 6 (p.186). Inasmuch as the viscosity of a gas increases with temperature, the viscosity near the wall for heat addition is greater than that near the center of the tube so that the gas is slowed down near the wall and accelerated in the center of the tube for heat addition to the gas. The opposite effect is produced for heat extraction from the gas. The variation of density across the tube apparently has a smaller effect on the velocity distributions than does the variation of viscosity.

2154 The changes in sharpness of the temperature profile at the tube center with heat addition and extraction, as indicated in figure 2, are in the same direction as those for velocity profiles because the thermal conductivity varies in the same way with temperature as does the viscosity. The shapes of velocity and temperature profiles for laminar flow, however, differ somewhat in contrast to turbulent flow where velocity and temperature profiles are similar (reference 7). Although equations (1) and (16) have the same form, the shear stress varies linearly with radius whereas the heat transfer per unit area does not, and hence the shape of the velocity and temperature profiles may be expected to differ. In turbulent flow the profiles are so flat that it makes little difference what assumptions are used for the variations of shear stress and heat transfer with radius.

Nusselt numbers and friction parameters. - Nusselt numbers and friction parameters are plotted as functions of t_0/t_b in figures 3 and 4, respectively. The Nusselt numbers were calculated from equations (32), (33), and (36) and the friction parameters were calculated from equations (38) and (39). It is seen from the figures that the parameters vary considerably with t_0/t_b when the fluid properties are evaluated at either the wall temperature or the bulk temperature. The effect of t_0/t_b on the parameters can, however, be nearly eliminated by evaluating the conductivity in the Nusselt number at $t_{-0.27} = -0.27(t_0-t_b) + t_b$ and by evaluating the viscosity in the friction parameter at $t_{0.58} = 0.58(t_0-t_b) + t_b$. The fluid properties in the Nusselt number must be evaluated at a different temperature than that at which the fluid properties in the friction parameter are evaluated, inasmuch as the velocity and temperature profiles are dissimilar and the bulk velocity and the bulk temperature are weighted differently (see equations (22) and (24)).

Laminar Flow of Liquid Metals

Velocity and temperature distributions. - The velocity and temperature distributions for laminar flow of liquid metals shown in figures 5 and 6 were calculated in the same way as were those for gases. The exponent d was found from the limited viscosity data available for liquid metals to have an average value of about -1.6 for lead, bismuth, and sodium potassium alloy at temperatures between 300° and 1500° F.

The changes in velocity-profile shapes shown in figure 5 are opposite to those for laminar flow of gases, because the variation with temperature of the viscosity of liquid metals is opposite to that of gases. It is seen in figure 6 that the effect of t_0/t_b on the shape of the temperature profile is slight for liquid metals, because the thermal conductivity is considered constant, so that any change in the temperature distribution must be caused by variation of viscosity. 2154

Nusselt numbers and friction parameters. - Nusselt numbers and friction parameters are plotted as functions of t_0/t_b in figures 7 and 8, respectively.

The curves in figure 7 show that the friction parameter can be made substantially independent of t_0/t_b by evaluating the viscosity at $t_{0.54} = 0.54(t_0-t_b) + t_b$. This temperature is close to the temperature used for evaluating the viscosity in the friction parameters for gases.

The curve in figure 8 shows the variation of Nusselt number with t_0/t_b for liquid metals. The effect of t_0/t_b on the Nusselt numbers cannot be eliminated as in the case of friction parameters, inasmuch as the thermal conductivity of liquid metals is assumed not to vary with temperature. The variation of Nusselt number with t_0/t_b is caused by the variation of viscosity with temperature and can be nearly eliminated by multiplying the Nusselt numbers in figure 8 by the empirical correction for liquids usually given in the literature $(\mu_0/\mu_b)^{0.14}$. (See, for example, reference 6, p. 190.) The fact that the Nusselt number for liquid metals varies with t_0/t_b indicates that Nusselt numbers for gases are probably affected by variation of viscosity as well as variation of conductivity. Evaluating the conductivity at a certain temperature in order to eliminate the effect of t_0/t_b seems to be only a convenient way of presenting the results.

Comparison of Laminar and Turbulent Flow of Gases

A comparison of laminar and turbulent flow of gases is given in figures 9 and 10.

In figure 9 are shown calculated laminar Nusselt numbers from the present investigation and calculated turbulent Nusselt numbers for a Prandtl number of 1 from reference 7. The laminar Nusselt number is independent of the Reynolds and Prandtl numbers, whereas the turbulent Nusselt number is dependent on them. In order to eliminate the effects of t_0/t_b, the fluid properties including the density in the Nusselt and Reynolds numbers are evaluated at the average of the wall and bulk temperatures $t_{0.5}$ for turbulent heat transfer and at a temperature between the bulk temperature and the temperature at the center of the tube $t_{-0.27}$ for laminar heat transfer. The fluid properties for heat transfer in the transition region should evidently be evaluated at temperatures between these two.

In figure 10, the laminar friction factors were calculated from equation (41) and the turbulent friction factors were taken from reference 7. The laminar friction factor f is independent of Prandtl number. (The laminar friction parameter δ is independent of both Reynolds and Prandtl numbers.) For friction factors, the fluid properties including the density for both laminar and turbulent flow are evaluated at temperatures close to the average of the wall and bulk temperatures.

SUMMARY OF RESULTS

The following results were obtained from an analytical investigation of laminar flow in tubes:

1. Variation of fluid properties across the tubes for laminar flow of gases caused a sharpening of the peak of the velocity and temperature profiles at the center of the tube for heat addition to the gas and a flattening of these profiles for heat extraction.

2. Variation of fluid properties across the tube for laminar flow of liquid metals caused a flattening of the velocity profile at the center of the tube for heat addition to the liquid metal and a sharpening of the profile for heat extraction. Variation of fluid properties had but a slight effect on the temperature profile for liquid metals.

3. The effects of ratio of wall to bulk temperature on Nusselt numbers and friction parameters for laminar flow with variable fluid properties could be eliminated by evaluating the fluid properties in these quantities at various temperatures in the fluid.

4. The velocity and temperature profiles, as well as Nusselt number and friction parameter, were independent of Reynolds and Prandtl number for fully developed laminar flow with fluid properties that varied across the tube.

National Advisory Committee for Aeronautics,
Lewis Flight Propulsion Laboratory,
Cleveland, Ohio, February 1, 1951.

REFERENCES

1. Jakob, Max: Heat Transfer. Vol. I. John Wiley & Sons, Inc., 1949, pp. 451-464.

2. von Kármán, Th., and Tsien, H. S.: Boundary Layer in Compressible Fluids. Jour. Aero. Sci., vol. 5, no. 6, April, 1938, pp. 227-232. 2154

3. Hantzsche, W., und Wendt, H.: Die laminare Grenzschicht der ebenen Platte mit und ohne Wärmeübergang unter Berücksichtigung der Kompressibilität. Jahrb 1942 der deutschen Luftfahrtforschung, S. 140-150.

4. Yamagata, Kiyosi: A Contribution to the Theory of Non-isothermal Laminar Flow of Fluids Inside a Straight Tube of Circular Cross Section. Memoirs Faculty Eng., Kyushu Imperial Univ. (Fukuka, Japan), vol. VIII, no. 6, 1940. (In English.)

5. Bakhmeteff, Boris A.: The Mechanics of Turbulent Flow. Princeton Univ. Press (Princeton), 1941, p. 2.

6. McAdams, William H.: Heat Transmission. McGraw-Hill Book Co., Inc., 2d ed., 1942.

7. Deissler, Robert G.: Analytical Investigation of Turbulent Flow in Smooth Tubes with Heat Transfer with Variable Fluid Properties for Prandtl Number of 1. NACA TN 2242, 1950.

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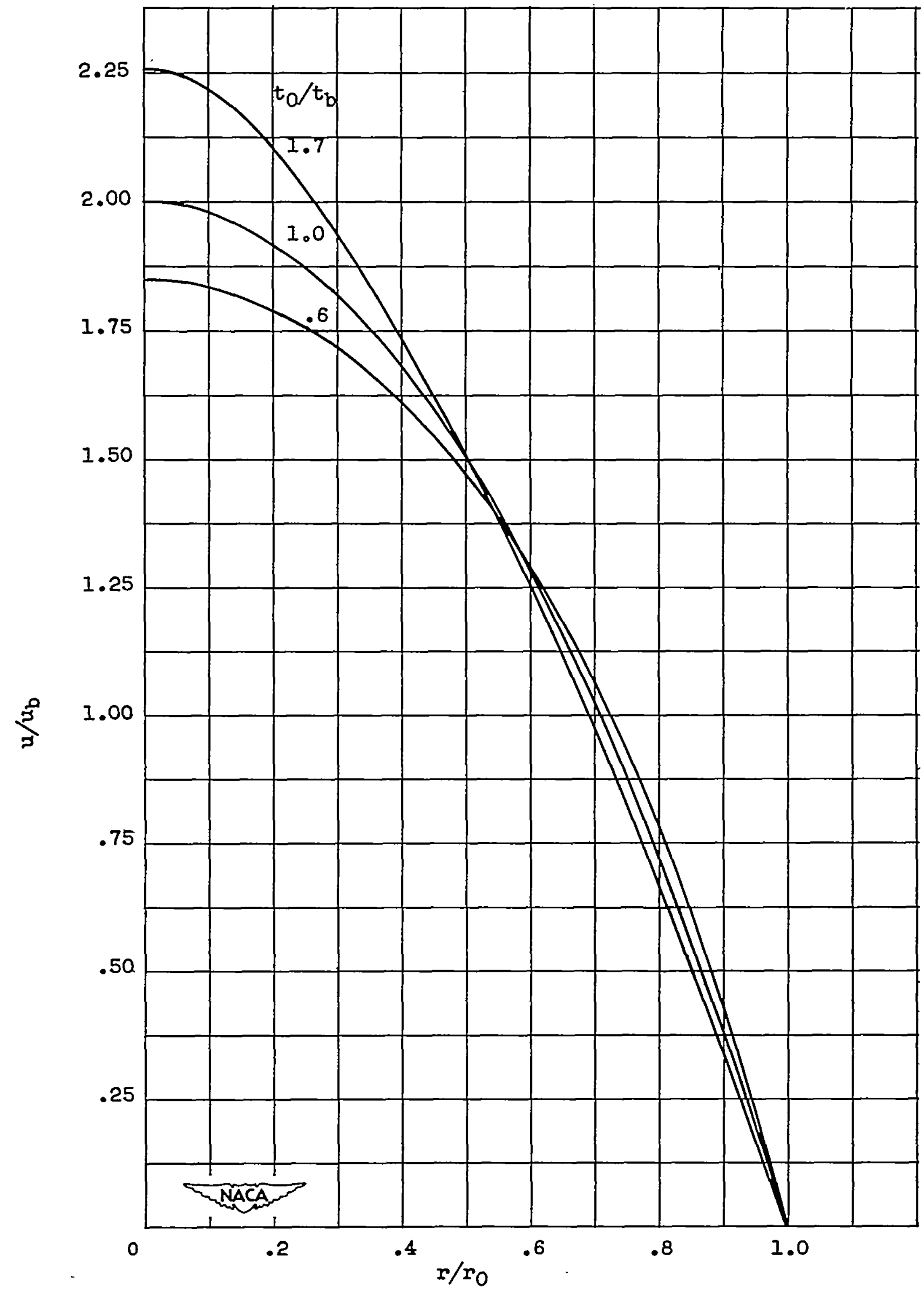

Figure 1. - Velocity distributions for laminar flow of gases in tubes.

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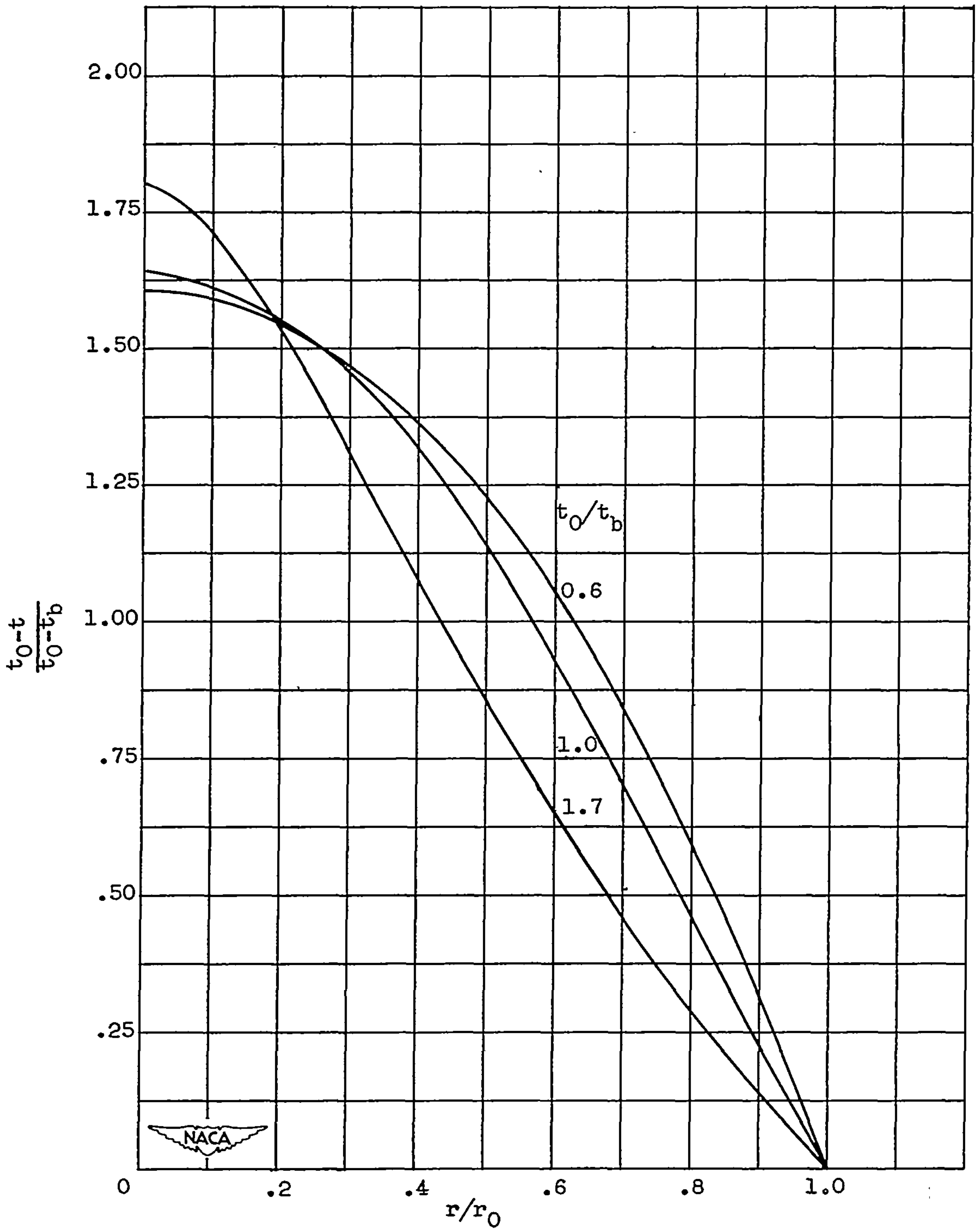

Figure 2. - Temperature distributions for laminar flow of gases in tubes.

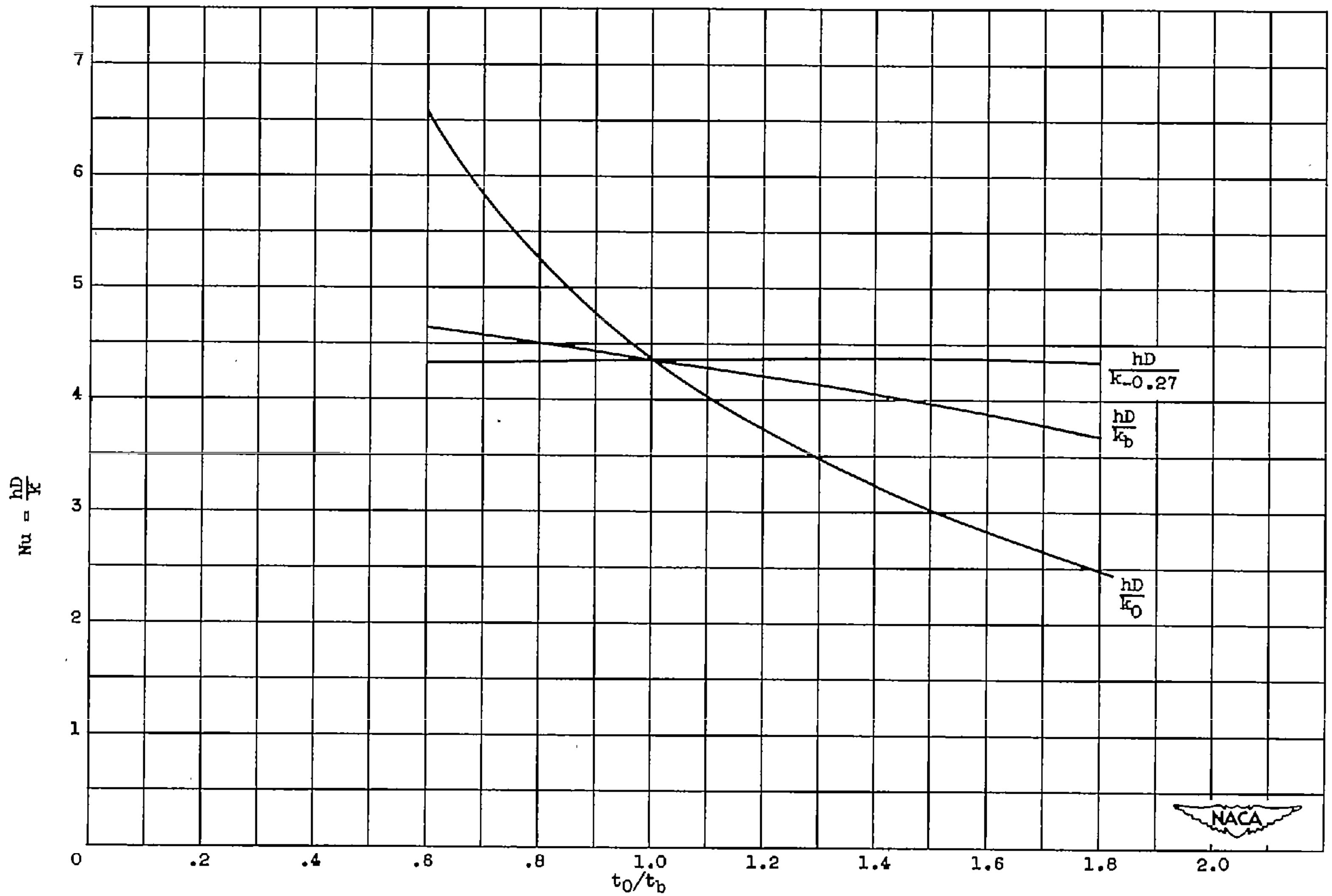

Figure 3. - Variation of Nusselt number with ratio of wall to bulk temperature for laminar flow of gases with conductivity evaluated at temperatures of $t_{-0.27} = -0.27\ (t_0 - t_b) + t_b$, t_b, and t_0.

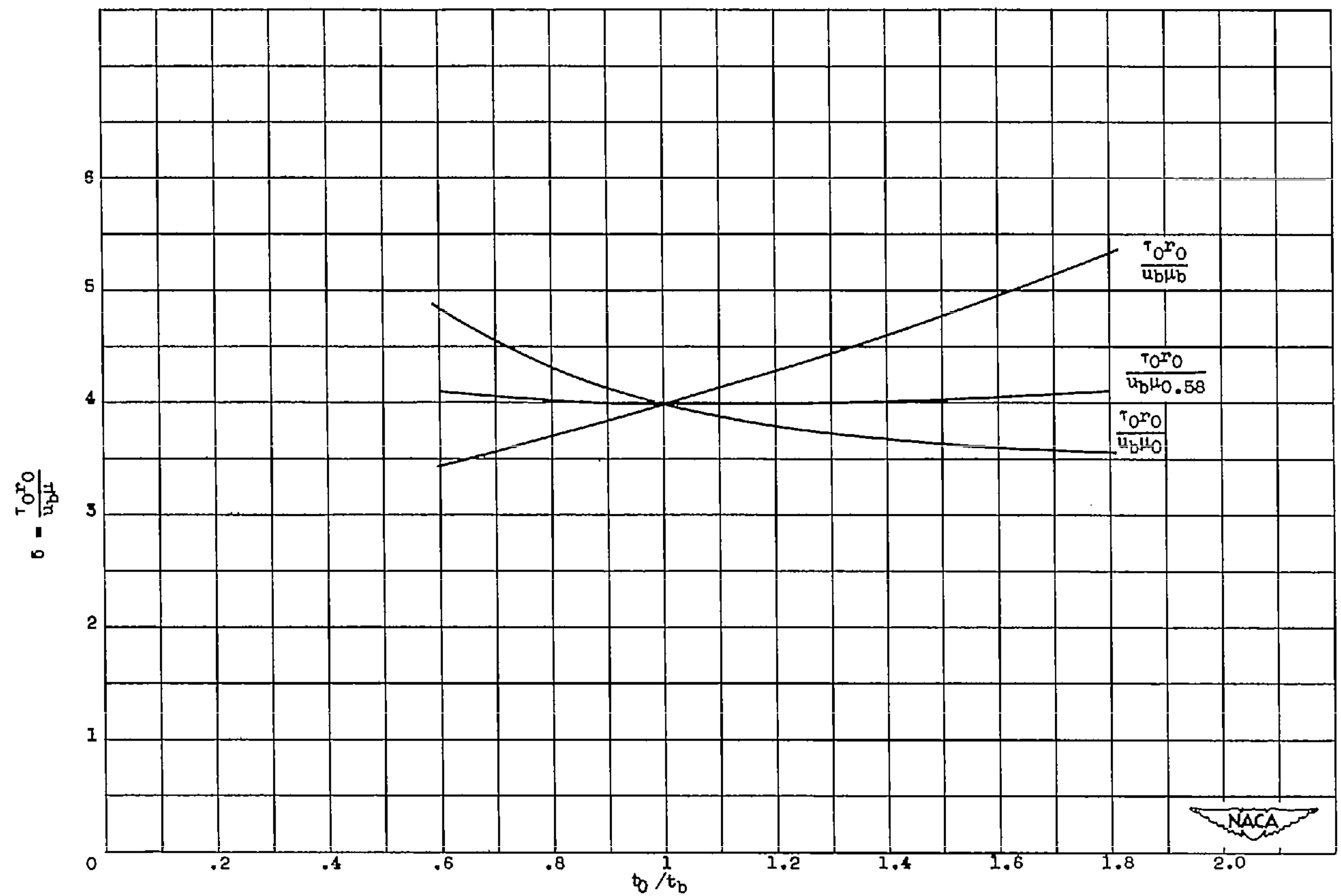

Figure 4. - Variation of friction parameter with ratio of wall to bulk temperature for laminar flow of gases with viscosity evaluated at temperatures of t_b, $t_{0.58} = 0.58\ (t_0 - t_b) + t_b$, and t_0.

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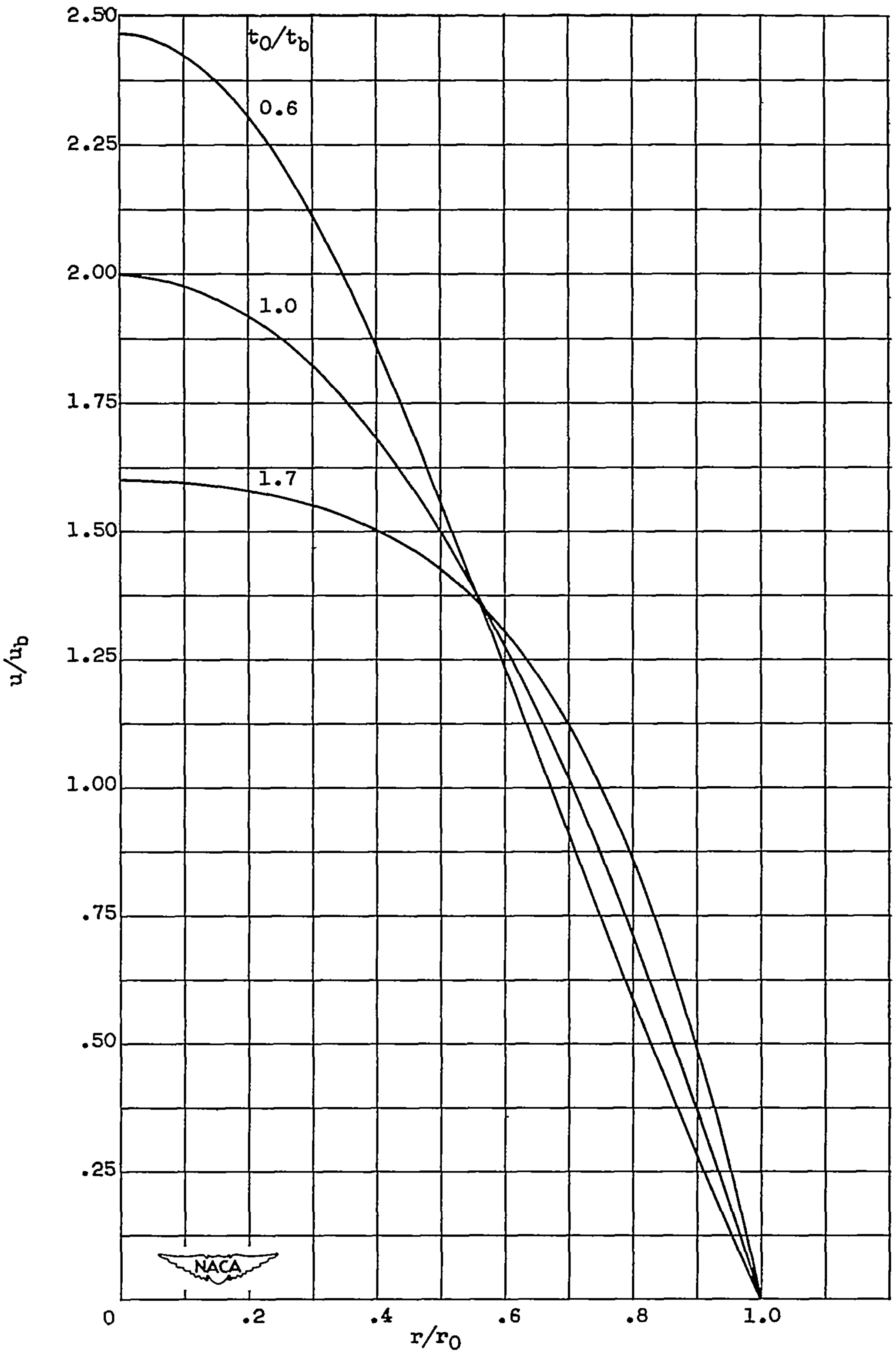

Figure 5. - Velocity distributions for laminar flow of liquid metals in tubes.

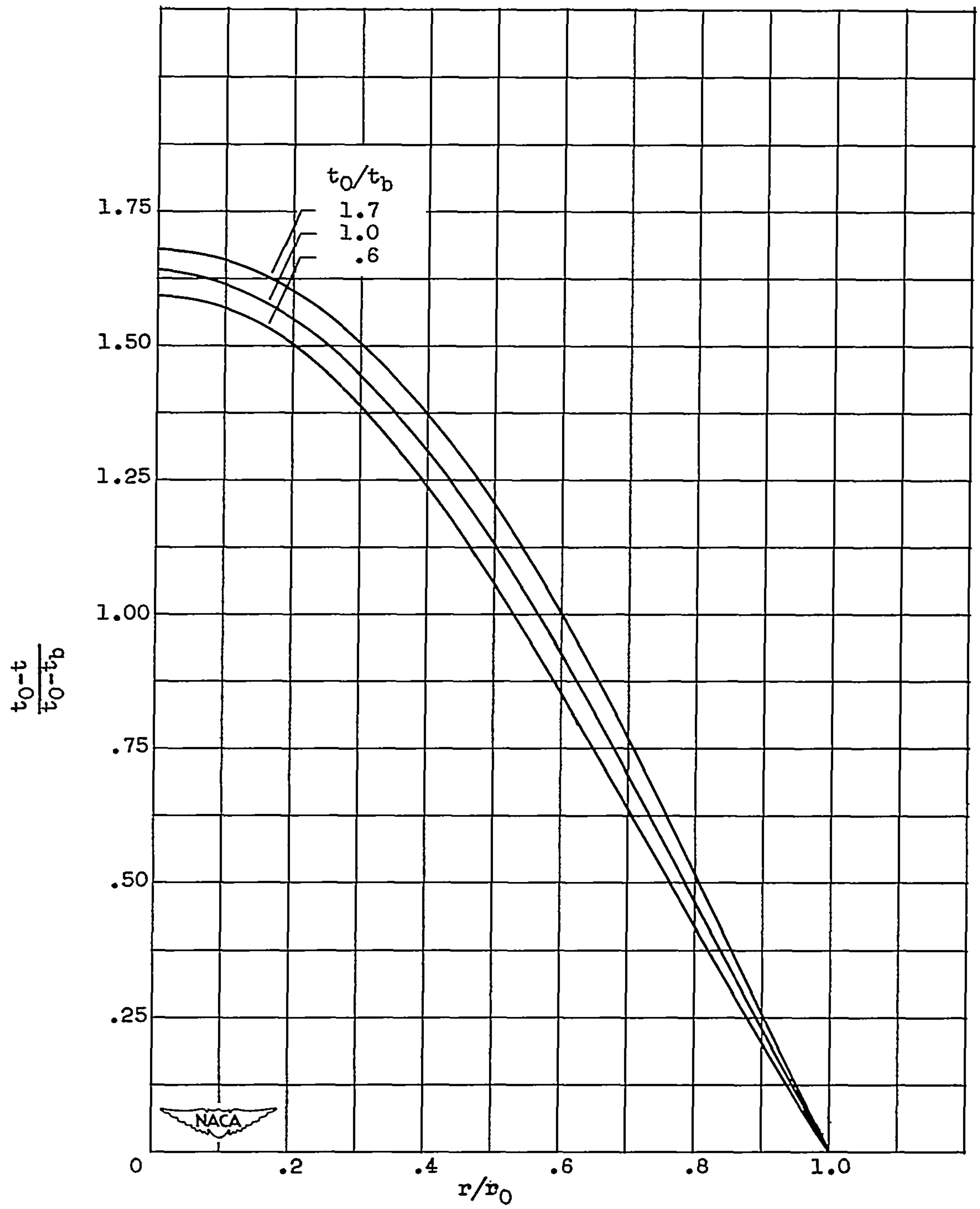

Figure 6. - Temperature distributions for laminar flow of liquid metals in tubes.

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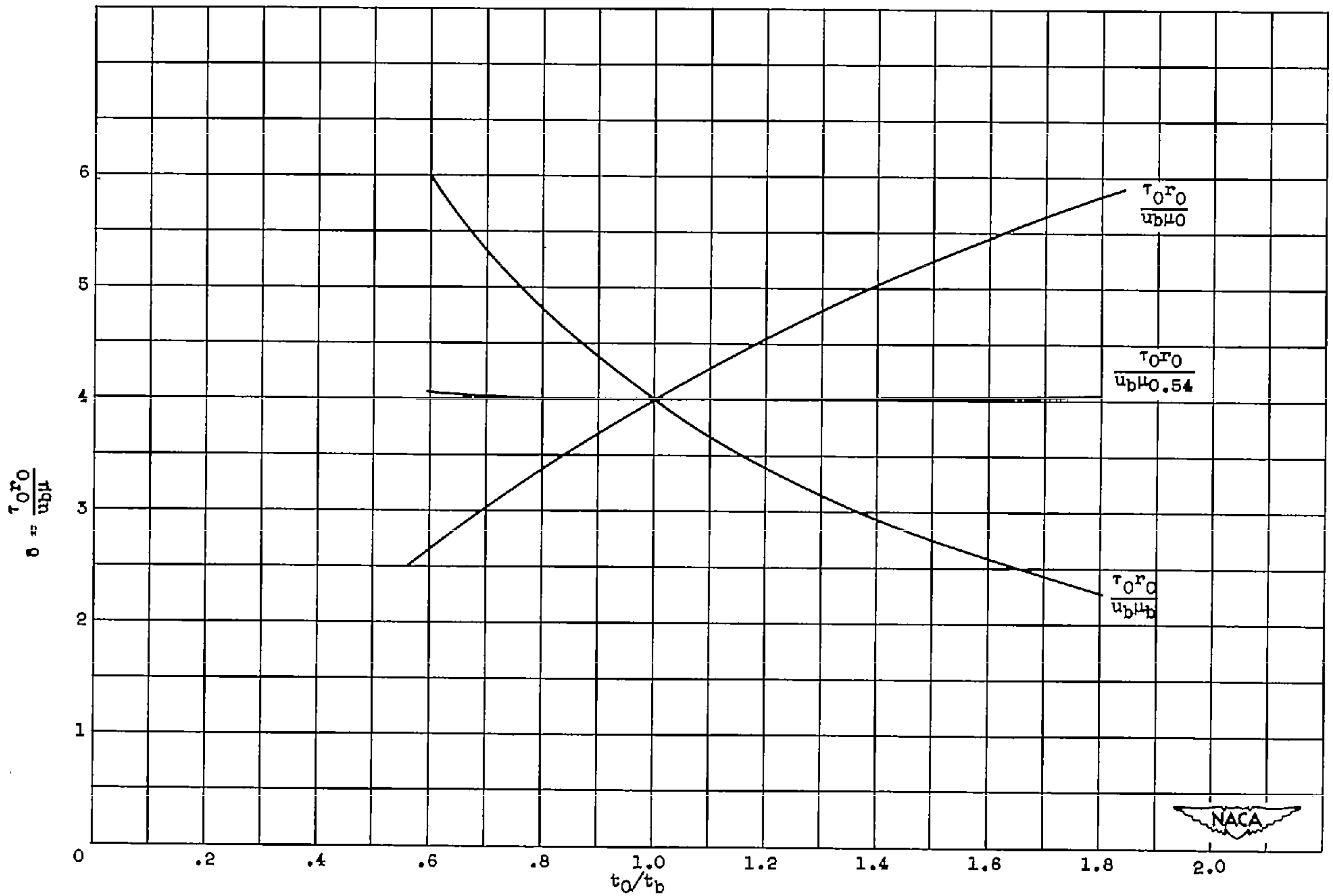

Figure 7. - Variation of friction parameter with ratio of wall to bulk temperature for laminar flow of liquid metals with viscosity evaluated at temperatures of t_0, $t_{0.54} = 0.54\ (t_0 - t_b) + t_b$, and t_b.

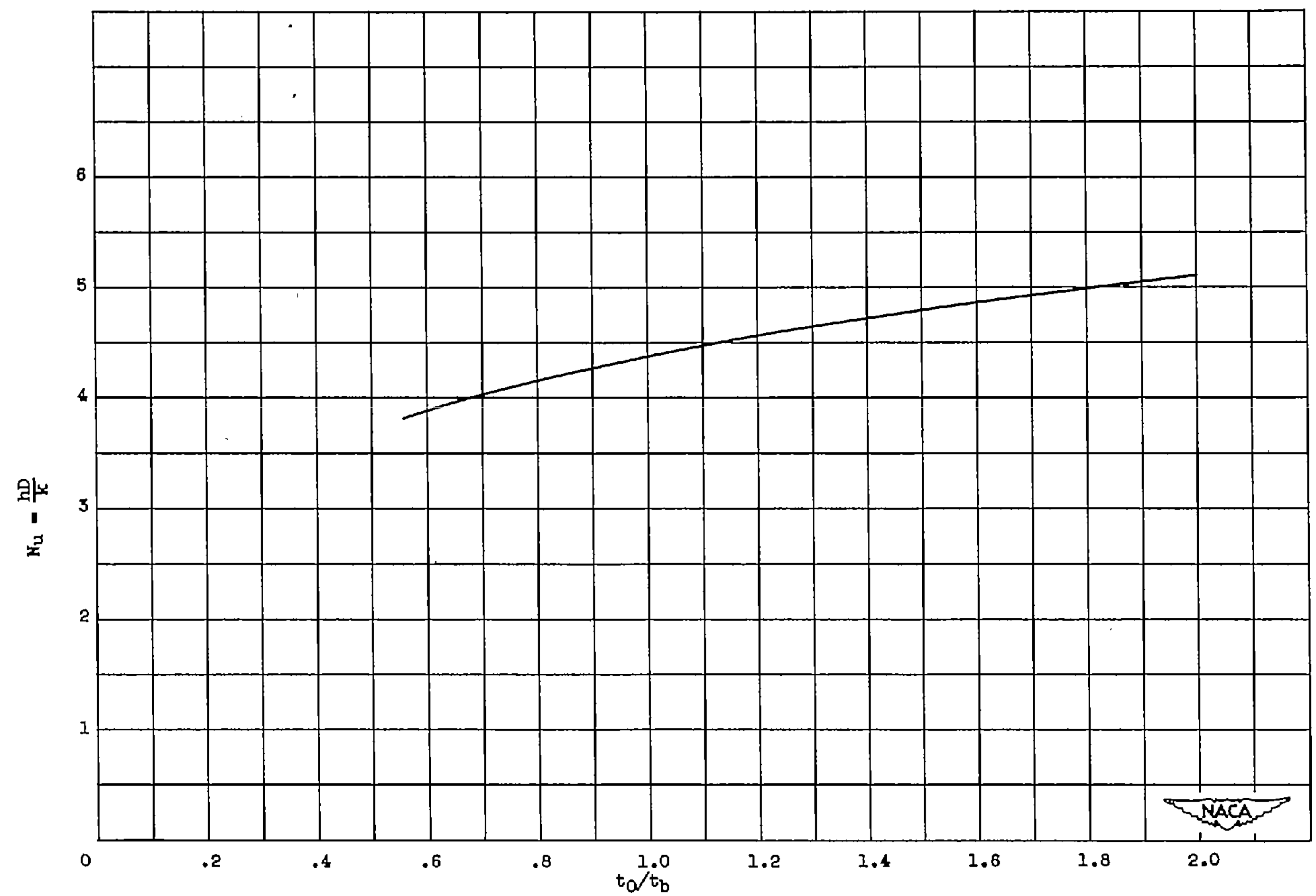

Figure 8. - Variation of Nusselt number with ratio of wall to bulk temperature for laminar flow of liquid metals with constant thermal conductivity.

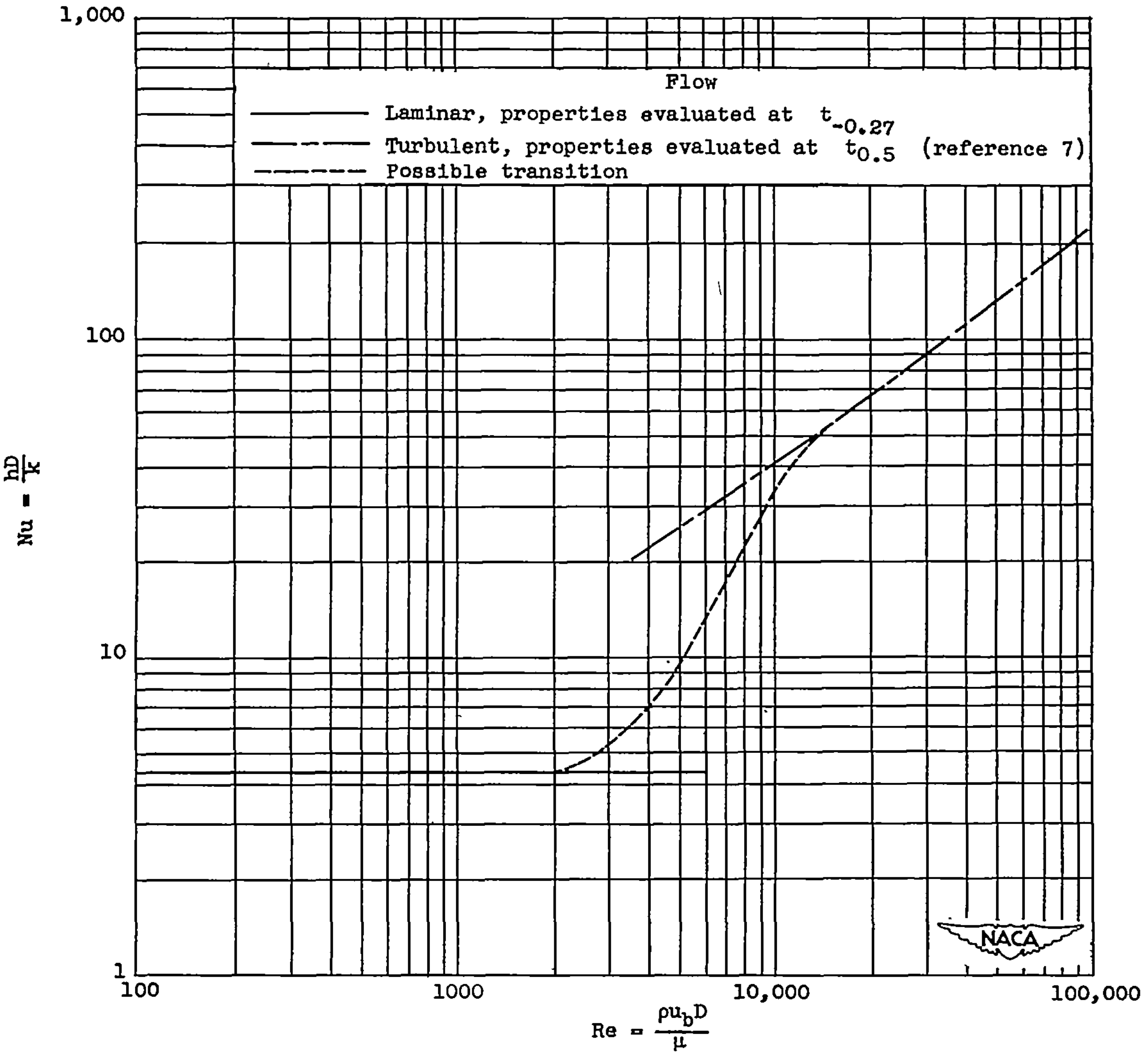

Figure 9. - Variation of Nusselt number with Reynolds number for flow of gases with fluid properties including density in Nusselt number and Reynolds number evaluated at temperatures of $t_{-0.27} = -0.27\ (t_0-t_b) + t_b$ for laminar heat transfer and at temperatures of $t_{0.5} = 0.5\ (t_0-t_b) + t_b$ for turbulent heat transfer at Prandtl number of 1.

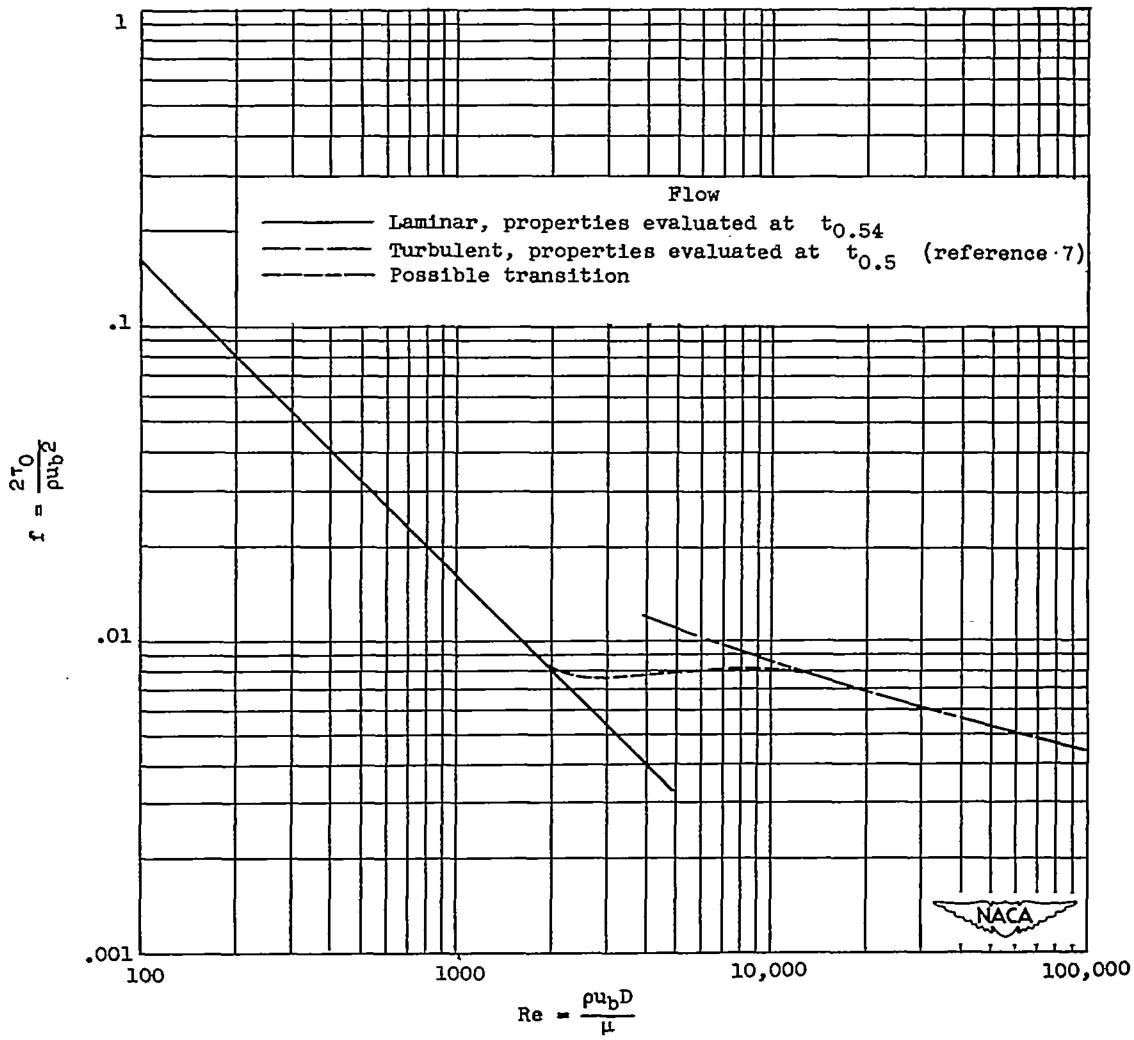

Figure 10. - Variation of friction factor with Reynolds number for flow of gases with fluid properties including density in friction factor and Reynolds number evaluated at temperatures of $t_{0.54} = 0.54\ (t_0 - t_b) + t_b$ for laminar flow and at temperatures of $t_{0.5} = 0.5\ (t_0 - t_b) + t_b$ for turbulent flow.